This Month’s Problems

- FE CIVIL Exam & Topics - Number of Questions
- Types of Calculators / For FE and PE Exams
- Technology Usage / Definite Integrals
- Technology Usage / Decimal & Binary Numbers
- Surveying / Bearings & Azimuths
- Surveying / Traverse Computations
- Centroids & Moments of Inertia
- Statics / Simple & Cantilever Beams
- Statics / Shear and Moment Diagrams
- Strength of Materials / Determinate Frames
- Strength of Materials / Determinate Structures
- Strength of Material / Flexural Stresses
- Reinforced Concrete Beam
The new Civil FE Computer-Based Test (CBT) consists of 110 multiple-choice questions (Each problem only one question) the examinee will have 6 hours to complete the test.

- Mathematics (Approx. 9 questions*)
- Probability and Statistics (5 questions)
- Computational Tools (5 questions)
- Ethics and Professional Practice (5 questions)
- Engineering Economics (5 questions)
- Statics (9 questions)
- Dynamics (5 questions)
- Mechanics of Materials (9 questions)
- Civil Engineering Materials (5 questions)
- Fluid Mechanics (5 questions)
- Hydraulics and Hydrologic Systems (10 questions)
- Structural Analysis (8 questions)
- Structural Design (8 questions)
- Geotechnical Engineering (12 questions)
- Transportation Engineering (10 questions)
- Environmental Engineering (8 questions)

* Here the number of questions are the average values taken from the NCEES Reference Handbook (Version 9.1 / Computer-Based Test)
To protect the integrity of FE/PE exams, NCEES limits the types of calculators you may bring to exam sites. The only calculator models acceptable for use during the 2016 exams are as follows:

**Casio:** All fx-115 models. Any Casio calculator must contain fx-115 in its model name. Examples of acceptable Casio fx-115 models include (but are not limited to):

- fx-115 MS
- fx-115 MS Plus
- fx-115 MS SR
- fx-115 ES
- fx-115 ES Plus

**Texas Instruments:** All TI-30X and TI-36X models. Any Texas Instruments calculator must contain either TI-30X or TI-36X in its model name. Examples of acceptable TI-30X and TI-36X models include (but are not limited to):

- TI-30Xa
- TI-30Xa SOLAR
- TI-30Xa SE
- TI-30XS Multiview
- TI-30X IIB
- TI-30X IIS
- TI-36X II
- TI-36X SOLAR
- TI-36X Pro

**Hewlett Packard:** The HP 33s and HP 35s models, but no others.
Frequently asked two Number Systems:

1- Decimal Number System (base 10)
2- Binary Number System (base 2)

In Decimal System 10 different digits are used to create any number, but in Binary System only 0s and 1s are used to create any number.

<table>
<thead>
<tr>
<th>DECIMAL</th>
<th>BINARY</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
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<td>15</td>
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<td>19</td>
<td>10011</td>
</tr>
<tr>
<td>25</td>
<td>11001</td>
</tr>
</tbody>
</table>
DEFINITE INTEGRALS
AREAS UNDER CURVES
Using CASIO FX-115 ES PLUS

Problem:

\[ f(x) = \left(\frac{2}{3}\right)^x \]

The area under the graph shown is most nearly:

(A) 5.06
(B) 4.25
(C) 2.15
(D) 1.37
Problem:

\[ I = \int_{0}^{3} x^2 \cdot (e^{x^2}) \, dx \]

The value of the definite integral shown above is most nearly:

(A) 20  
(B) 3986  
(C) 1112  
(D) 11432

Comments:

At first sight, this problem seems highly complex, but it is not. You can easily get the correct answer in exactly 15 seconds. Yes, in 15 seconds!
What can you do with your CASIO FX-115 ES PLUS?

Enter the equation into the calculator

Just hit the \( = \) key and wait 15 seconds

Answer = 11432.353
**Problem:** (Bearings & Azimuths)

A closed traverse is given as shown in the figure. Using the listed internal angles and the bearing for the side $AB$ answer the following:

1. the north azimuth of $AB$ (deg-min-sec) is most nearly
   - (A) $10^\circ 20' 30''$
   - (B) $20^\circ 00' 00''$
   - (C) $30^\circ 40' 00''$
   - (D) $100^\circ 00' 00''$

   Azim ($AB$) = ?

2. the north azimuth of $BC$ (deg-min-sec) is most nearly
   - (A) $238^\circ 43' 34''$
   - (B) $222^\circ 05' 41''$
   - (C) $190^\circ 12' 28''$
   - (D) $167^\circ 05' 41''$

   Azim ($BC$) = ?

3. the north azimuth of $CD$ (deg-min-sec) is most nearly
   - (A) $163^\circ 43' 18''$
   - (B) $112^\circ 25' 50''$
   - (C) $61^\circ 03' 18''$
   - (D) $81^\circ 43' 18''$

   Azim ($CD$) = ?

The figure is drawn not to scale
Problem: (Bearings & Azimuths)

<table>
<thead>
<tr>
<th>Course</th>
<th>Length (ft)</th>
<th>Bearing (deg-min-sec)</th>
<th>Azimuth (deg-min-sec)</th>
<th>Y coor.</th>
<th>X coor.</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>632.46</td>
<td>N 20-00-00 E</td>
<td>20-00-00</td>
<td>+100.00</td>
<td>+100.00</td>
</tr>
<tr>
<td>BC</td>
<td>640.31</td>
<td>S 12-54-19 E</td>
<td>167-05-41</td>
<td>+694.25</td>
<td>+316.19</td>
</tr>
<tr>
<td>CD</td>
<td>650.85</td>
<td>N 61-03-18 E</td>
<td>61-03-18</td>
<td>+70.04</td>
<td>+459.07</td>
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<tr>
<td>DE</td>
<td>531.51</td>
<td>S 02-45-03 E</td>
<td>177-14-57</td>
<td>+384.96</td>
<td>+1028.49</td>
</tr>
<tr>
<td>EF</td>
<td>721.11</td>
<td>S 72-07-30 W</td>
<td>252-07-30</td>
<td>-146.00</td>
<td>+1053.89</td>
</tr>
<tr>
<td>FA</td>
<td>538.52</td>
<td>N 29-45-49 W</td>
<td>330-14-11</td>
<td>-367.42</td>
<td>+367.44</td>
</tr>
</tbody>
</table>

Error of closure = 0.852 feet
Precision = 1 in 4360
Area = 485,000 sq.ft
Area = 11.134 acres
BEARINGS & AZIMUTHS
SURVEYING

Problem: (Bearings & Azimuths)

\[ A = 130^\circ 14' 11'' \]
\[ B = 32^\circ 54' 19'' \]
\[ C = 286^\circ 02' 23'' \]
\[ D = 63^\circ 48' 21'' \]
\[ E = 105^\circ 07' 27'' \]
\[ F = 101^\circ 53' 19'' \]

Bearing of the course \( AB \): S 80° 00' 00'' E

A closed traverse is given as shown in the figure. Using the listed internal angles and the bearing for the side \( AB \) answer the following:

(1) the north azimuth of \( AB \) (deg-min-sec) is most nearly

(A) \( 142^\circ 09' 424'' \)
(B) \( 159^\circ 33' 52'' \)
(C) \( 120^\circ 10' 20'' \)
(D) \( 100^\circ 00' 00'' \)

Azim (\( AB \)) = ?

(2) the north azimuth of \( BC \) (deg-min-sec) is most nearly

(A) \( 265^\circ 13' 54'' \)
(B) \( 247^\circ 05' 41'' \)
(C) \( 230^\circ 40' 20'' \)
(D) \( 162^\circ 30' 10'' \)

Azim (BC) = ?

(3) the north azimuth of \( CD \) (deg-min-sec) is most nearly

(A) \( 213^\circ 20' 58'' \)
(B) \( 180^\circ 25' 50'' \)
(C) \( 159^\circ 36' 30'' \)
(D) \( 141^\circ 03' 18'' \)

Azim (CD) = ?

The figure is drawn not to scale
Problem: (Bearings & Azimuths)

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{Course} & \text{Length (ft)} & \text{Bearing (deg-min-sec)} & \text{Azimuth (deg-min-sec)} & \text{Y coor.} & \text{X coor.} \\
\hline
AB & 632.46 & S 80-00-00 E & 100-00-00 & +100.00 & +100.00 \\
CD & 650.85 & S 38-56-42 E & 141-03-18 & -258.81 & +132.85 \\
DE & 531.51 & S 77-14-57 W & 257-14-57 & -764.90 & +541.86 \\
FA & 538.52 & N 50-14-11 E & 050-14-11 & -244.55 & -313.88 \\
\hline
\end{array}
\]

Error of closure = 0.855 feet
Precision = 1 in 4340
Area = 485,000 sq. ft
Area = 11.134 acres
**Problem:** (Bearings & Azimuths)

![Diagram of a polygon with bearings and angles]

Bearing of the course \( AB \) : S 01\(^\circ\) 01' 01" E

A closed traverse is given as shown in the figure. Using the listed internal angles and the bearing for the side \( AB \) answer the following:

1. **the north azimuth of \( AB \) (deg-min-sec) is most nearly**
   - (A) 142° 29' 424"
   - (B) 159° 33' 52"
   - (C) 161° 10' 12"
   - (D) 178° 58' 59"
   
   **Azim \( AB \) = ?**

2. **the north azimuth of \( BC \) (deg-min-sec) is most nearly**
   - (A) 265° 13' 54"
   - (B) 257° 35' 49"
   - (C) 238° 49' 28"
   - (D) 162° 36' 18"
   
   **Azim \( BC \) = ?**

3. **the north azimuth of \( CD \) (deg-min-sec) is most nearly**
   - (A) 153° 26' 54"
   - (B) 281° 15' 59"
   - (C) 259° 16' 57"
   - (D) 114° 27' 12"
   
   **Azim \( CD \) = ?**

**The figure is drawn not to scale**
Course : Surveying
Coordinator : Zeytinci

Traverse Computation:

<table>
<thead>
<tr>
<th>Point</th>
<th>Internal Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>091-29-10</td>
</tr>
<tr>
<td>B</td>
<td>101-23-10</td>
</tr>
<tr>
<td>C</td>
<td>156-19-50</td>
</tr>
<tr>
<td>D</td>
<td>196-35-15</td>
</tr>
<tr>
<td>E</td>
<td>085-02-45</td>
</tr>
<tr>
<td>F</td>
<td>089-09-50</td>
</tr>
</tbody>
</table>

Bearing of AB:
SOI-01-01 E

Notation:
91-29-10
91 degrees
29 minutes
10 seconds

A closed traverse is given as shown. Bearing of the side AB and all internal angles are listed as shown in the table.

a-) Determine the course bearings of the other sides.
b-) Determine the course azimuths of all sides.

List all your answers in the table given below:

<table>
<thead>
<tr>
<th>Course</th>
<th>Bearings</th>
<th>Azimuths</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>SOI-01-01 E</td>
<td>178-58-59 W</td>
</tr>
<tr>
<td>BC</td>
<td>S 77-35-49 W</td>
<td>257-35-49</td>
</tr>
<tr>
<td>CD</td>
<td>N 78-44-01 W</td>
<td>281-15-59</td>
</tr>
<tr>
<td>DE</td>
<td>S 84-40-44 W</td>
<td>264-40-44</td>
</tr>
<tr>
<td>EF</td>
<td>N 90-22-01 W</td>
<td>255-37-59</td>
</tr>
<tr>
<td>FA</td>
<td>N 90-28-09 W</td>
<td>90-28-09</td>
</tr>
</tbody>
</table>

A13
SOI-01-01 E

Excellent
**Problem:**

A simply supported beam is loaded as shown. Knowing that point “C” is the mid-point of the beam, answer the following questions:

1- the magnitude of the shear force (kips) at the mid-point of the beam is most nearly:

   - (A) 0
   - (B) 12
   - (C) 16
   - (D) 20

   \[ V_c = ? \]

2- the maximum bending moment (k-ft) in the beam is most nearly:

   - (A) 86
   - (B) 64
   - (C) 48
   - (D) 120

   \[ M_{\text{max}} = ? \]
The cantilever beam is loaded as shown in the figure. Using the given loads answer the following questions:

1. Knowing that the moment at support A is 72 ft-kips determine the value of the moment at B is most nearly

   (A) 25
   (B) 14
   (C) 12
   (D) 9

2. The magnitude of the bending moment (ft-kips) at the mid-point of the beam (6-feet from A) is most nearly, $M_{mid\ point}$

   (A) 18
   (B) 24
   (C) 29
   (D) 45
BEAMS WITH INTERNAL HINGES  

DOMAIN: STATICS

Two beams are connected with an internal pin at C as shown. Using the given loads and the support conditions, determine the following:

(1) the vertical support reaction (kips) at support B

(A) 4.50
(B) 4.75
(C) 5.00
(D) 6.00

(2) the bending moment (ft-kips) at support A

(A) 238
(B) 214
(C) 144
(D) 124

(3) the bending moment (ft-kips) at point D

(A) 48
(B) 45
(C) 36
(D) 30
SIMPLE BEAM SUPPORT REACTIONS
SHORTCUT FORMULAS

1

\[ A_y = P \left( \frac{b}{L} \right) \]
\[ B_y = P \left( \frac{a}{L} \right) \]

(FBD)

2

\[ A_y = \frac{M}{L} \]
\[ B_y = \frac{M}{L} \]

(FBD)

FE/PE EXAM

BM-15
ZEYTINCI
FALL 2014
SIMPLE BEAM SUPPORT REACTIONS
SUPERPOSITION METHOD

Example-1

\[
P = 40 \text{ kips}\]

\[
A_y = P \left( \frac{b}{L} \right) = 40 \left( \frac{5}{20} \right) = 10 \text{ kips} \uparrow
\]

\[
B_y = P \left( \frac{a}{L} \right) = 40 \left( \frac{15}{20} \right) = 30 \text{ kips} \uparrow
\]

Example-2

\[
M = 60 \text{ kips}\]

\[
A_y = \frac{M}{L} = \frac{60}{12} = 5 \text{ kips} \uparrow
\]

\[
B_y = \frac{M}{L} = \frac{60}{12} = 5 \text{ kips} \downarrow
\]

Example-3

\[
P = 70 \text{ kips}\]

\[
A_y = \frac{M}{L} = \frac{40}{10} = 4 \text{ kips} \uparrow
\]

\[
B_y = \frac{M}{L} = \frac{40}{10} = 4 \text{ kips} \downarrow
\]
Problem: (Beam Support Reactions / Superposition Method)

Determine the support reactions using the superposition (shortcut) method.

\[ A_y = 3.17 \text{ kips} + 2.5 \text{ kips} + 1.33 \text{ kips} = 7.0 \text{ kips} \]
\[ B_y = 0.83 \text{ kips} + 2.5 \text{ kips} + 6.67 \text{ kips} = 10.0 \text{ kips} \]
**Problem: (Cantilever Beams)**

The cantilever beam is loaded as shown in the figure. Using the given loads and the support condition, answer the following questions:

(1) The vertical support reaction (kN) at the fixed support, $A_y$

(A) 80  
(B) 75  
(C) 60  
(D) 50

(2) The bending moment (kN.m) at the fixed support, $M_A$

(A) 230  
(B) 340  
(C) 425  
(D) 560

(3) The moment (kN.m) at $C$, the mid-point of $AB$, is most nearly, $M_C$

(A) 110  
(B) 125  
(C) 138  
(D) 155

(4) The shear force (kN) at the mid-point of $AB$ is most nearly

(A) 42.5  
(B) 50.0  
(C) 67.5  
(D) 70.0
**Problem:** (Cantilever Beams)

The cantilever beam is loaded as shown in the figure. Using the given loads and the support condition, answer the following questions:

1. The vertical support reaction (kN) at the fixed support, $A_y$
   - (A) 140
   - (B) 125
   - (C) 110
   - (D) 100

2. The bending moment (kN.m) at the fixed support, $M_A$
   - (A) 290
   - (B) 390
   - (C) 465
   - (D) 520

3. The moment (kN.m) at $C$, the mid-point of $AB$, is most nearly, $M_C$
   - (A) 515
   - (B) 420
   - (C) 338
   - (D) 210

4. The shear force (kN) at the mid-point of $AB$ is most nearly
   - (A) 42.5
   - (B) 50.0
   - (C) 60.5
   - (D) 70.0
**Problem:** (Cantilever Beams)

The cantilever beam is loaded as shown in the figure. Using the given loads and the support condition, answer the following questions:

(1) The horizontal and vertical support reactions (kN) at the fixed support, $A_x, A_y$

(A) 5.46 and 38
(B) 6.68 and 42
(C) 7.12 and 64
(D) 9.24 and 82

(2) The bending moment (kN.m) at the fixed support, $M_A$

(A) 416
(B) 314
(C) 225
(D) 176

(3) The moment (kN.m) at C, the mid-point of $AB$, is most nearly, $M_C$

(A) 93
(B) 113
(C) 128
(D) 136

(4) The shear force (kN) at the mid-point of $AB$ is most nearly

(A) 22.5
(B) 24.6
(C) 37.5
(D) 42.0
**Problem:** (Cantilever Beams)

A cantilever beam is loaded as shown in the figure. Using the given loads and the support condition, answer the following questions:

(1) The vertical support reaction (kN) at the fixed support, $A_y$

(A) 69  
(B) 78  
(C) 87  
(D) 99

(2) The bending moment (kN.m) at the fixed support, $M_A$

(A) 125  
(B) 185  
(C) 205  
(D) 225

(3) The horizontal support reaction (kN) at the fixed support, $A_x$

(A) 15.0  
(B) 18.0  
(C) 21.5  
(D) 28.0

(4) The bending moment (kN.m) at the mid-point of $AB$ is most nearly

(A) 94.5  
(B) 108.4  
(C) 122.5  
(D) 148.7
**Problem:** (Cantilever Beams)

A cantilever beam is loaded as shown in the figure. Using the given loads and support condition, answer the following questions:

(1) The vertical support reaction (kN) at the fixed support, $A_y$

(A) 88  
(B) 105  
(C) 113  
(D) 128

(2) The bending moment (kN.m) at the fixed support, $M_A$

(A) 510  
(B) 414  
(C) 325  
(D) 284

(3) The moment (kN.m) at $C$, the mid-point of $AB$ is most nearly, $M_C$

(A) 90  
(B) 110  
(C) 122  
(D) 134

(4) The shear force (kN) at the mid-point of $AB$ is most nearly

(A) 21.5  
(B) 34.0  
(C) 47.5  
(D) 52.5
**STRENGTH OF MATERIALS**

**FLEXURE**

**Problem:** (Bending)

A cantilever beam with a rectangular tube section is loaded as shown. Using the listed data, answer the following questions:

1. the allowable stress (MPa) with a Factor of Safety of 3 is ($\sigma_{\text{all}}$)
   - (A) 80
   - (B) 100
   - (C) 150
   - (D) 200

2. the the bending moment (kN.m) for which the F.S. will be 3.0 ($M$)
   - (A) 4.0
   - (B) 5.0
   - (C) 6.0
   - (D) 8.5

3. the radius of curvature in meters is most nearly ($\rho$)
   - (A) 35.0
   - (B) 42.0
   - (C) 54.5
   - (D) 60.6

4. the maximum strain is most nearly ($\varepsilon_{\text{max}}$)
   - (A) $1.25 \times 10^{-3}$
   - (B) $1.43 \times 10^{-3}$
   - (C) $1.56 \times 10^{-3}$
   - (D) $1.63 \times 10^{-3}$
**Problem:** (Centroid / MOI / Bending Stresses)

An overhanging beam is loaded as shown in the figure. Using the given cross-section, answer the following questions:

1. the location (in.) of the centroid $\bar{y}$ is most nearly
   - (A) 3.90
   - (B) 4.80
   - (C) 5.50
   - (D) 6.00

2. the moment of inertia about the horiz. centroidal axis $I_{cx}$ (in$^4$)
   - (A) 820.05
   - (B) 714.48
   - (C) 605.14
   - (D) 523.07

3. the maximum tensile stress (ksi) is most nearly
   - (A) 4.41
   - (B) 3.18
   - (C) 2.86
   - (D) 1.95

4. the maximum compressive stress (ksi) is most nearly
   - (A) 1.67
   - (B) 2.98
   - (C) 3.20
   - (D) 4.36
Problem: (Centroid / Bending Stresses)

An overhanging beam is loaded as shown in the figure. Using the given cross-section, answer the following questions:

(1) the location (in.) of the centroid is most nearly (\( \bar{y} \))

(A) 4.0
(B) 4.5
(C) 5.0
(D) 6.0

(2) the moment of inertia about the horiz. centroidal axis \( I_{cx} \) (in\(^4\))

(A) 689
(B) 592
(C) 522
(D) 489

(3) the maximum tensile stress (ksi) is most nearly (\( \sigma_{tensile} \))

(A) 3.41
(B) 4.18
(C) 5.52
(D) 5.71

(4) the maximum compressive stress (ksi) is most nearly (\( \sigma_{comp} \))

(A) 6.40
(B) 5.55
(C) 4.20
(D) 3.55
Problem: (Centroid / MOI / V & M Diagrams / Bending Stresses)

An overhanging beam is loaded as shown in the figure. Using the given section properties, answer the following questions:

(1) the distance (in.) of the centroid of the cross-section, \( \bar{y} \)
   
   (A) 3.50
   (B) 3.75
   (C) 4.00
   (D) 4.25

(2) the moment of inertia (in\(^4\)) about the horiz. centroidal axis, \( I_{cx} \)
   
   (A) 415
   (B) 375
   (C) 333
   (D) 296

(3) the maximum tensile stress (psi) in the cross-section is most nearly
   
   (A) 4805
   (B) 4120
   (C) 3660
   (D) 3030

(4) the maximum compressive stress (psi) in the cross-section is most nearly
   
   (A) 3540
   (B) 3780
   (C) 4215
   (D) 2695
Problem: (Centroid / MOI / V & M Diagrams / Bending Stresses)

An overhanging beam is loaded as shown in the figure. Using the given cross-section, answer the following questions:

(1) the location (in.) of the centroid $\bar{y}$ is most nearly

(A) 4.05
(B) 4.50
(C) 5.65
(D) 6.40

(2) the moment of inertia about the horiz. centroidal axis $I_{cx}$ (in$^4$)

(A) 889.45
(B) 790.40
(C) 622.15
(D) 589.30

(3) the maximum tensile stress (ksi) is most nearly

(A) 4.41
(B) 3.18
(C) 2.43
(D) 1.95

(4) the maximum compressive stress (ksi) is most nearly

(A) 1.67
(B) 2.33
(C) 3.20
(D) 4.36
A simply supported beam is made of A36 steel and loaded as shown in the figure. Knowing that there is no inplane buckling and using the listed data, answer the following questions:

(1) the magnitude of the maximum bending moment (k-ft) is most nearly:

(A) 105  
(B) 110  
(C) 125  
(D) 145

(2) Using the new NCEES Reference Manual (V.9.2) the most economical W-shape is most nearly:

(A) W 10 x 54  
(B) W 12 x 40  
(C) W 14 x 43  
(D) W 16 x 40
(a) DETERMINE THE SUPPORT REACTIONS.

\[ \sum M_B = 0 \]
\[ 3k(6) + 6k(18) + 3k(20) - 24A_y = 0 \]
\[ 180 + 96 + 60 - 24A_y = 0 \]
\[ 24A_y = 336 \]
\[ A_y = 14k \]

\[ \sum M_A = 0 \]
\[ 3k(4) + 6k(8) + 30k(18) - 24B_y = 0 \]
\[ 12 + 48 + 540 - 24B_y = 0 \]
\[ 24B_y = 600 \]
\[ B_y = 25k \]

CHECK: \[ A_y + B_y = \frac{3 + 6 + 30}{14 + 25} = \frac{39}{39} = 1 \] OK
(b) DRAW THE SHEAR FORCE DIAGRAM.
(c) DRAW THE MOMENT DIAGRAM.

\[ M_{\text{max}} = \frac{1}{2} (10' \cdot 25k) \]
\[ M_{\text{max}} = 125 \text{ k-ft} \]
\[ S = \frac{M_{\text{max}}}{f_{\text{allow}}} \]
\[ S = \frac{(125 \text{ k-ft})(12 \text{ in.})}{24 \text{ k/in}^2} = 62.5 \text{ in}^3 \]

(d) DESIGN THE BEAM.

\[ W \ 16 \times 40 \ (S = 64.7) \]
\[ W \ 14 \times 43 \ (S = 62.6) \]
\[ W \ 12 \times 53 \ (S = 70.6) \]
\[ W \ 10 \times 60 \ (S = 66.7) \]
\[ W \ 10 \times 54 \ (S = 60.0) \]

EXCELLENT SOLUTION!
Using the matrices given above, answer the following questions:

(1) the determinant of the matrix \( A \) is most nearly

(A) \( 5 \)
(B) \( -3 \)
(C) \( -7 \)
(D) \( 4 \)

(2) the matrix product of \( A \) and \( B \) is most nearly, \( A*B \)

(A) \[
A*B = \begin{bmatrix}
-2 & 11 & 9 \\
-8 & -2 & 2
\end{bmatrix}
\]

(B) \[
A*B = \begin{bmatrix}
-1 & 7 & -4 \\
0 & -5 & -2
\end{bmatrix}
\]

(C) \[
A*B = \begin{bmatrix}
3 & 12 & 6 \\
-1 & 7 & 2
\end{bmatrix}
\]

(D) \[
A*B = \begin{bmatrix}
2 & 11 & 9 \\
-8 & 5 & -1
\end{bmatrix}
\]
Problem: (Matrix Algebra)

\[
A = \begin{bmatrix}
3 & -3 & 2 \\
2 & 5 & -3 \\
3 & -1 & 1 \\
\end{bmatrix}
\]

Using the matrix given above, answer the following questions:

1. the determinant of the above matrix is most nearly

   (A) 4 
   (B) 5 
   (C) 6 
   (D) 8 

2. the inverse of matrix \( A \) is most nearly, \( (A^{-1}) \)

   (A) \[
   A^{-1} = \begin{bmatrix}
1/4 & 1/4 & -1/4 \\
10/4 & 5/4 & 13/4 \\
-10/4 & -4/4 & 11/4 \\
\end{bmatrix}
   \]
   
   (B) \[
   A^{-1} = \begin{bmatrix}
2/3 & 1/3 & -1/6 \\
-11/3 & -7/3 & 13/3 \\
-17/3 & -8/3 & 32/3 \\
\end{bmatrix}
   \]
   
   (C) \[
   A^{-1} = \begin{bmatrix}
2/5 & 1/5 & 1/5 \\
11/5 & 3/5 & 13/5 \\
-17/5 & -6/5 & 21/5 \\
\end{bmatrix}
   \]
   
   (D) \[
   A^{-1} = \begin{bmatrix}
2/5 & 1/5 & -1/5 \\
-11/5 & -3/5 & 13/5 \\
-17/5 & -6/5 & 21/5 \\
\end{bmatrix}
   \]
**Problem: (Matrix Algebra)**

\[
B = \begin{bmatrix}
1 & 2 & 0 \\
2 & -5 & -2 \\
-1 & 3 & 1
\end{bmatrix}
\]

A square matrix \([B]\) is given above. Answer the following questions:

(1) the determinant of matrix \([B]\) is most nearly, \(\text{det} [B]\)

(A) 1  
(B) 2  
(C) -3  
(D) -1

(2) the inverse of matrix \([B]\) is most nearly, \((B^{-1})\)

(A) \(B^{-1} = \begin{bmatrix}
1 & 2 & 4 \\
0 & 13 & 1 \\
4 & -5 & 3
\end{bmatrix}\)

(B) \(B^{-1} = \begin{bmatrix}
10 & -2 & 7 \\
0 & 13 & 9 \\
2 & -8 & 2
\end{bmatrix}\)

(C) \(B^{-1} = \begin{bmatrix}
1 & -2 & -4 \\
0 & -3 & -8 \\
3 & -9 & 2
\end{bmatrix}\)

(D) \(B^{-1} = \begin{bmatrix}
1 & -2 & -4 \\
0 & 1 & 2 \\
1 & -5 & -9
\end{bmatrix}\)

*IN NEXT THE NEWSLETTER*  
*WE WILL SHOW YOU HOW TO GET ANSWERS ONLY IN 2B SECONDS!*
Using the matrices given above, answer the following questions:

(1) the determinant of the matrix \( A \) is most nearly

(A) 4
(B) -3
(C) -2
(D) 1

(2) the matrix product of \( A \) and \( B \) is most nearly, \( A*B \)

(A) \[
A*B = \begin{bmatrix}
-4 & 12 & -10 \\
2 & -5 & -2
\end{bmatrix}
\]

(B) \[
A*B = \begin{bmatrix}
-1 & 12 & -4 \\
0 & -5 & -2
\end{bmatrix}
\]

(C) \[
A*B = \begin{bmatrix}
-4 & 19 & 6 \\
-1 & 7 & 2
\end{bmatrix}
\]

(D) \[
A*B = \begin{bmatrix}
-2 & 11 & -20 \\
2 & 13 & 7
\end{bmatrix}
\]
**IMPORTANT FORMULAS**

**Problem: (Rectangle)**

![Rectangle Diagram]

C: Centroid

Origin of axes at C

(a) Area of the rectangle \( A = ? \)
(b) Centroid of the rectangle \( \bar{x} = ? \)
(c) Moment of inertia about the x axis \( I_{cx} = ? \)
(d) Moment of inertia about the y axis \( I_{cy} = ? \)

**Formulas:**

\[
A = bh \\
\overline{y} = \frac{h}{2} \\
I_{cx} = \frac{bh^3}{12} \\
I_{cy} = \frac{h^3}{12} 
\]

**Example:**

\[
A = bh = (8)(4) = 32 \text{ in.}^2 \\
\overline{y} = \frac{h}{2} = \frac{4}{2} = 2 \text{ in.} \\
I_{cx} = \frac{bh^3}{12} = \frac{(8)(4)^3}{12} = 42.67 \text{ in.}^4 \\
I_{cy} = \frac{h^3}{12} = \frac{(4)^3}{12} = 170.67 \text{ in.}^4 
\]

**Problem: (Triangular Area)**

![Isosceles Triangle Diagram]

C: Centroid

\( x, y \): Centroidal axes

(a) Area of the triangle \( A = ? \)
(b) Centroid of the triangle \( \bar{x} = ? \)
(c) Moment of inertia about the x axis \( I_{cx} = ? \)
(d) Moment of inertia about the y axis \( I_{cy} = ? \)

**Formulas:**

\[
A = \frac{bh}{2} \\
\overline{y} = \frac{h}{3} \\
I_{cx} = \frac{bh^3}{36} \\
I_{cy} = \frac{h^3}{48} 
\]

**Example:**

\[
A = \frac{bh}{2} = \frac{(8)(6)}{2} = 24 \text{ in.}^2 \\
\overline{y} = \frac{h}{3} = \frac{6}{3} = 2 \text{ in.} \\
I_{cx} = \frac{bh^3}{36} = \frac{(8)(6)^3}{36} = 48.00 \text{ in.}^4 \\
I_{cy} = \frac{h^3}{48} = \frac{(6)(8)^3}{48} = 64.00 \text{ in.}^4 
\]

**Problem: (Half Circle)**

![Half Circle Diagram]

(a) Area of the semicircle \( A = ? \)
(b) Centroid of the semicircle \( \bar{y} = ? \)
(c) Moment of inertia about the x axis \( I_{cx} = ? \)
(d) Moment of inertia about the y axis \( I_{cy} = ? \)

**Solution:**

\[
A = \frac{\pi r^2}{2} \\
\overline{y} = \frac{4r}{3\pi} \\
I_{cx} = \frac{r^4}{72\pi}, \quad \left(9\pi^2 - 64\right) \\
I_{cy} \approx 0.109757 \cdot r^4 \\
I_{cx} \approx 0.109757 \cdot r^4 \\
I_{cy} = \frac{\pi r^4}{8} 
\]

**Problem: (Trapezoidal Area)**

![Trapezoid Diagram]

C: Centroid

\( x, y \): Centroidal axes

Isosceles Trapezoid

(a) Area of the trapezoid \( A = ? \)
(b) Centroid of the trapezoid \( \bar{x} = ? \)
(c) Moment of inertia about the x axis \( I_{cx} = ? \)
(d) Moment of inertia about the y axis \( I_{cy} = ? \)

**Formulas:**

\[
A = \frac{h}{2} \left( a + b \right) \\
\overline{y} = \frac{h}{3} \left( \frac{2a + b}{a + b} \right) \\
I_{cx} = \frac{h^3}{36} \cdot \frac{a^2 + 4ab + b^2}{a + b} \\
I_{cy} = \frac{h}{48} \left( a + b \right) \left( a^2 + b^2 \right) \\
I_{cy} = \frac{h^3}{12} \left( 3a + b \right) 
\]

**CENT-111**

ZEYTINCI

SPRING 2014
**Problem:** (Parallel Axis Theorem)

![Diagram](image)

(a) Moment of inertia about the x axis \( (I_{cx} = ?) \)
(b) Moment of inertia about the (1-1) axis \( (I_{1-1} = ?) \)

**Formulas:**

\[
I_{cx} = \frac{bh^3}{12}
\]

\[
I_{1-1} = I_{cx} + A \cdot d^2
\]

\[
I_{cx} = \text{Moment of inertia about the centroidal x axis}
\]
\[
I_{1-1} = \text{Moment of inertia about axis (1-1)}
\]
\[
A = \text{Area (} A = b.h \text{)}
\]
\[
d = \text{Distance from the centroid to the axis (1-1)}
\]

**Example:**

![Diagram](image)

\[
I_{cx} = \frac{bh^3}{12} = \frac{(6)(3)^3}{12} = 13.50 \text{ in.}^4
\]
\[
A = bh = (6)(3) = 18 \text{ in.}^2
\]
\[
d = 4 + \frac{3}{2} = 5.5 \text{ in.}
\]
\[
I_{1-1} = I_{cx} + A \cdot d^2 = 13.50 + (18)(5.5)^2 = 558 \text{ in.}^4
\]

\[
I_{1-1} = 558 \text{ in.}^4
\]
**Problem:** (Deflections)

A cantilever beam is loaded as shown. The beam weight is included in the uniform load. Using the given cross-section and the modulus of elasticity answer the following questions:

1. the max. deflection (in.) of the beam is most nearly ($\delta_B$)
   
   (A) 0.0367 
   (B) 0.4929 
   (C) 0.6525 
   (D) 1.4456 

2. the slope (rad.) at the free end of the beam is most nearly ($\theta_B$)
   
   (A) 0.06458 
   (B) 0.03782 
   (C) 0.00415 
   (D) 0.00596 

3. the moment at the support is most nearly ($M_A$)
   
   (A) 75 
   (B) 83 
   (C) 90 
   (D) 105
Problem: (Deflections)

A simple beam is loaded as shown. The beam weight is included in the uniform load. Using the given cross-section and the modulus of elasticity answer the following questions:

(1) the max. deflection (in.) of the beam is most nearly \( \delta_{\text{max}} \)

(A) 0.75
(B) 1.00
(C) 1.91
(D) 2.50

(2) the slope (radians) at support A is most nearly \( \theta_A \)

(A) 1.226
(B) 0.425
(C) 0.055
(D) 0.021

(3) the maximum moment (k.ft) in the beam is most nearly \( M_{\text{max}} \)

(A) 174
(B) 250
(C) 285
(D) 316

\( E = 29 \times 10^6 \text{ psi} \)
**Problem:** (Beam Deflections)

A cantilever beam is loaded as shown. The beam weight is included in the uniform load. Using the listed data and the modulus of elasticity answer the following questions:

1. The max. deflection (in.) of the beam is most nearly ($\delta_B$)
   
   (A) 0.5426  
   (B) 0.6550  
   (C) 0.0877  
   (D) 0.0915  

2. The max. slope (rad.) at the free end is most nearly ($\theta_B$)
   
   (A) 0.02153  
   (B) 0.01554  
   (C) 0.00777  
   (D) 0.00655  

3. The bending moment (k.ft) at the support is most nearly ($M_A$)
   
   (A) 155  
   (B) 182  
   (C) 190  
   (D) 220
Problem: (Deflections)

A composite beam is made of two W 12 x 40 welded together. The beam weight is included in the uniform load. Using the given beam dimensions and loads answer the following questions:

(1) the max. moment (ft.kips) in the beam is most nearly

(A) 126
(B) 165
(C) 225
(D) 300

(2) the moment of inertia (in.\(^4\)) about the horizontal centroidal axis is most nearly \(I_{cx}\)

(A) 307
(B) 515
(C) 614
(D) 720

(3) the max. deflection (in.) of the beam is most nearly

(A) 0.245
(B) 0.363
(C) 0.478
(D) 0.594

(4) the slope (radians) at the left support is most nearly

(A) 0.00985
(B) 0.00768
(C) 0.00643
(D) 0.02250
SHEAR FORCE AND MOMENT DIAGRAMS

1  
\[ \begin{align*} 
A & \quad 9k \\
8\text{ ft} & \quad 4\text{ ft} \\
9k & \quad (FBD) \\
3k & \quad (V) \\
6k & \quad (M) \\
24\text{ kft} & 
\end{align*} \]

2  
\[ \begin{align*} 
A & \quad 12k \\
5\text{ ft} & \quad 4\text{ ft} \\
8k & \quad (FBD) \\
12k & \quad (V) \\
8k & \quad (M) \\
45\text{ kft} & \\
33\text{ kft} & 
\end{align*} \]

3  
\[ \begin{align*} 
A & \quad 4k \\
4\text{ ft} & \quad 4\text{ ft} \\
5k & \quad 8k \\
4k & \quad (FBD) \\
5k & \quad (V) \\
8k & \quad (M) \\
7.5k & \quad + \\
7.5k & \quad + \\
3.5k & \quad - \\
1.5k & \quad - \\
9.5k & \quad + \\
30\text{ kft} & \\
44\text{ kft} & \\
38\text{ kft} & 
\end{align*} \]

4  
\[ \begin{align*} 
A & \quad 9k \\
4\text{ ft} & \quad 4\text{ ft} \\
5k & \quad 8k \\
6k & \quad (FBD) \\
5k & \quad (V) \\
8k & \quad (M) \\
9k & \quad + \\
3k & \quad + \\
2k & \quad - \\
10k & \quad - \\
36\text{ kft} & \\
48\text{ kft} & \\
40\text{ kft} & 
\end{align*} \]
SHEAR AND MOMENT DIAGRAMS

5

2k/ft

12 ft

12 k

Zero Shear

6 ft

36 ft.k

12 k

(V)

(M)

6

2k/ft

(2k/ft)

A

B

13 kips

6.5 ft

42.25 ft.k

3k

40 ft.k

7k

13 kips

Zero Shear

(M)

(k, ft)

7

6k

8k

4k

5ft

4ft

4ft

4ft

A

B

5k

13k

1k

4k

1k

20k.ft

16k.ft

20k.ft

(FBD)

(V)

(k)

(M)

(k-ft)

8

3k/ft

6k

A

B

14k

16k

14k

Zero Shear

6k

4.67ft

32

67

16.00

24.00 k.ft

(FBD)

(V)

(k, lb)

(M)

(k, ft)
SHEAR AND MOMENT DIAGRAMS

9

2.0 kips/ft

A

B

15.0 ft

6.0 ft

2.0 kips/ft

(FBD)

A_y = 12.60 kips

B_y = 29.40 kips

12.60 kips

12.00 kips

x = 6.30 ft

17.40 kips

M_{max} = + 39.69 ft-kip

36.00 ft-kip

(V)

(M)

10

2 k/ft

16 ft-kip

8 kip

A

B

8 ft

4 ft

4 ft

4 ft

2 k/ft

16 ft-kip

8 kip

(FBD)

11 kip

13 kip

11 kip

5.5 ft

5 kip

30.25 ft-kip

4 ft-kip

12 ft-kip

32 ft-kip

(V)

(M)

11

5 kips

4 kips

2.0 kips/ft

(FBD)

A

B

5.0 ft

5.0 ft

10.0 ft

5 kips

4 kips

2.0 kips/ft

A_y = 36.0 kips

B_y = 13.0 kips

21.0 kips

11.0 k

7.0 k

15.0 k

x = 6.5 ft

13.0 k

M_{max} = + 42.25 ft-kip

50.0 ft-kip

(V)

(M)

12

6 k/ft

4 k

2 k/ft

8 k

1.0 k/ft

5 kip

A

B

4 ft

6 ft

4 ft

4 ft

2.0 ft

4 ft

6 k/ft

2 k/ft

4 k

8 k

1.0 k/ft

5 kip

(FBD)

A_y = 23 kips

B_y = 20 kips

15 kips

3 k

9 kips

5 kips

8 k

1 k

9 k

11 kips

32 ft-kip

28 ft-kip

22 ft-kip

28 ft-kip

(V)

(M)
Shear Force and Bending Moment Diagrams

Support A: Hinge
Support B: Roller

The shear force diagram of this beam is most nearly:

(FBD)
SHEAR FORCE & BENDING MOMENT DIAGRAMS

1

2

3

4
Shear Force and Moment Diagrams

The shear force diagram of the beam is most nearly:

(A)  

(B)  

(C)  

(D)
Problem: Shear Force and Moment Diagrams

The moment diagram of this beam is most nearly:

(A)

(B)

(C)

(D)
The dimensions, loading and support conditions of a determinate frame are given as shown in the figure:

(1) the magnitude of the support reaction (kips) at point B

- (A) 8.0
- (B) 6.4
- (C) 5.5
- (D) 4.0

(2) the magnitude of the axial force (kips) at point A

- (A) 18.4
- (B) 14.6
- (C) 12.8
- (D) 10.6

(3) the magnitude of the shear force (kips) at point A is most nearly:

- (A) 8.6
- (B) 10.4
- (C) 12.8
- (D) 16.5
Solution by Dr. Vagelis Plevris

Answers:
(1) D (4.0)
(2) C (12.8)
(3) B (10.4)

Support Reactions

Axial Force and Shear Force of member AC at point A

Element End Forces

Software BEAM.2D by ENGILAB
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FUNDAMENTALS OF ENGINEERING
MECHANICS OF SOLIDS
Internal Forces in Determinate Frames

Problem:

The dimensions, loading and support conditions of a determinate frame are given as shown in the figure:

(1) the magnitude of the support reaction (kips) at point B
   (A) 24
   (B) 18
   (C) 14
   (D) 12

(2) the magnitude of the axial force (kips) at point A
   (A) 16.4
   (B) 14.4
   (C) 12.8
   (D) 10.6

(3) the magnitude of the max. bending moment (ft-kips) in member CB is most nearly:
   (A) 68.6
   (B) 72.4
   (C) 81.0
   (D) 96.5
Model

Model Equilibrium
\[ \Sigma F_X = 0.00 \]
\[ \Sigma F_Y = 0.00 \]
\[ \Sigma M = 0.00 \]

Support Reactions

Answers:
(1) B (18)
(2) B (14.4)
(3) C (81.0)

Axial Force Diagram

Bending Moment Diagram

Software BEAM.2D by ENGILAB

www.engilab.com
Problem:

The dimensions and support conditions of a determinate frame are given as shown. Assuming that the vertical load \( P = 10 \text{ kip} \) is applied at \( C \), answer the following questions:

(1) the \textit{bending moment diagram} of this frame is composed of:

- (A) Two triangles
- (B) Two rectangles
- (C) One triangle and two rectangles
- (D) One rectangle and one triangle

(2) the \textit{shear force diagram} of this frame is composed of:

- (A) Two triangles
- (B) Two rectangles
- (C) One triangle and two rectangles
- (D) One rectangle and one triangle

Support \( A \) : Pin
Support \( B \) : Roller
Answers:
(1) A (Two triangles)
(2) B (Two rectangles)
Internal Forces in Determinate Frames

**Problem:**

The dimensions and support conditions of a determinate frame are given as shown. Assuming that the vertical load $P=10$ kip is applied at $C$, answer the following questions:

(1) the magnitude of the *bending moment* (ft-kips) at point $C$ is most nearly:

(A) 25  
(B) 35  
(C) 50  
(D) 60

(2) the magnitude of the *shear force* (kips) in the inclined members is most nearly:

(A) 3.4  
(B) 4.6  
(C) 5.8  
(D) 6.6
Shear Force Diagram

Bending Moment Diagram

Shear Force Diagram

Answers:
(1) D (60)
(2) B (4.6)

Software BEAM.2D by ENGILAB
www.engilab.com
TRANSFORMED AREA METHOD
REINFORCED CONCRETE BEAM

\[ w = 2.0 \text{ k/ft} \]

\( n = 10 \)
(Modular Ratio = 10)

A reinforced concrete beam is loaded as shown in the figure. Assume the weight of the beam is included in the uniform load and the sections have cracked. Using the Transformed Area Method (TAM), answer the following:

1. the max. compressive stress (psi) in concrete is most nearly
   (A) 2400
   (B) 2250
   (C) 2140
   (D) 1850

\( f_c = ? \)

2. the max. tensile stress (psi) in steel is most nearly
   (A) 32,600
   (B) 29,400
   (C) 26,350
   (D) 24,500

\( f_s = ? \)
$W_{12} \times 50$

Nominal Depth (12 in.)

Weight
50 lb/ft

- $A = \text{Area} = 14.6 \text{ in}^2$
- $d = \text{Depth} = 12.2 \text{ in.}$
- $b_f = \text{Flange Width} = 8.08 \text{ in.}$
- $t_f = \text{Flange Thickness} = 0.640 \text{ in.}$
- $t_w = \text{Web Thickness} = 0.370 \text{ in.}$
- $I_x = \text{Moment of Inertia} = 391 \text{ in}^4$
- $I_y = \text{Moment of Inertia} = 56.3 \text{ in}^4$
- $S_x = \text{Section Modulus (x-x)} = 64.2 \text{ in}^3$
- $r_x = \text{Radius of Gyration} = 5.18 \text{ in}$
- Major axis or strong axis = (x-x)
- Minor axis or weak axis = (y-y)
"Basically being a PE means that you are at the top of your game, top of your profession. They don’t just hand that out to anybody." -- Randal E. Riebel, P.E.

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